Improved Wind Power Prediction
– final report –

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Contents

Preface

This report summarize the results of the project *Improved Wind Power Prediction* which has been conducted over the period 1. July, 2005 to 31. December, 2006. The project were financially supported by the Danish utilities PSO fund (FU-5766) and with Informatics and Mathematical Modelling, Technical University of Denmark as the entity responsible for the project. The further participants in the project were Risø National Laboratory (part of Technical University of Denmark as of January 1, 2007), Elsam Kraft A/S and Energi E2 A/S (now DONG Energy A/S). The transmission companies, first Elkraft System and then Energinet.dk followed the progress and attended meetings. The project budget was 1.107 million DKr, of which 0.890 million DKr was a grant under the PSO rules.

The following publications were produced as part of the project:

Thordarson et al. (2007): Thordarson F. Ö., Madsen H., Nielsen H.A. Optimal combined wind power forecasts using exogenous variables. Informatics and Mathematical Modelling, Technical University of Denmark, Technical Report (PSO-FU5766) No. 17, 2007.

Kotwa et al. (2007): Kotwa E., Nielsen H.A., Madsen H., Vlasova J. Spatio-temporal modelling of short-term wind power prediction errors. Informatics and Mathematical Modelling, Technical University of Denmark, Technical Report (PSO-FU5766) No. 18, 2007.

Møller et al. (2007b): Møller J.K., Nielsen H.A., Madsen H., Combined Forecast and Quantile Regression Informatics and Mathematical Modelling, Technical University of Denmark, Technical Report (PSO-FU5766) No. 19, 2007.

Nielsen et al. (2007b): Nielsen H.Aa., Nielsen T.S., Madsen H., San Isidro M.J., and Marti I. Optimal combination of wind power forecasts. *Wind Energy*, 2007c. Accepted.

Møller et al. (2008): Møller J.K., Nielsen H.A., Madsen H. Time adaptive quantile regression. Computational Statistics and Data Analysis, 52:1292-1303, 2008.

Christiansen and Nielsen (2006): Christiansen L.E., Nielsen T.S. Shifting NWPs in time to improve short time predictions. Technical Report (PSO-FU5766), IMM-DTU, 2006.

Møller et al. (2007a): Møller J.K., Nielsen H.A., Madsen H. Algorithms for Adaptive Quantile Regression – and a Matlab Implementation, Technical Report (PSO-FU5766), IMM-DTU, 2007.

Summary

In this PSO project several methods have been studied for improving both the accuracy and the estimation of the uncertainty of the wind power predictions, and in general very promising results are obtained:

Spatial-temporal correlation

Today the wind power prediction for a region (e.g. the Jutland-Fuen area) is found by upscaling the single wind farm power predictions and the spatial-temporal correlations between the single wind farm are not taking into account. The purpose of this part of the project is to make a pilot study of the potentials of taking into account the spatial-temporal correlations.

In this project it is demonstrated that a ridgid transformation of the prediction errors in order to account for phase errors of the meteorological prediction only leads to a minor improvement of the wind power predictions.

On the other hand, by using a non-linear stochastic model between prediction errors at different locations a huge potential for obtaining improved wind power predictions exists. The results found in this project, where only data from the Jutland-Fuen area is considered, show that around 50% of the prediction errors can be explained by a spatial-temporal extension of the advanced prediction systems used today, like WPPT. Since only a limited geographical area is considered, the results are valid for a one-hour predictions, but in cases where data from a larger area (e.g. also UK og Netherlands) is available, it is expected that also for larger horisonts a very large improvement will be seen.

Uncertainty of wind power predictions

It has been demonstrated that the value of having a reliable estimate of the uncertainty is of the same order as the value of have a high quality (state-of-art) prediction system like WPPT.

In this project a new and very promising method for estimating the uncertainty for wind power predictions is developed. The method is called **Adaptive Quantile Regression**, and it is seen that this approach can be linked to the WPPT forecasts rather easily, and hereby a reliable estimated of the uncertainty of the wind power prediction will be obtained.

Another method for providing reliable estimates of the uncertainty of the wind power pre-

diction is a previously developed method based on statistically calibrated meteorological ensembles. The advantages of the newly developed method based on adaptive quantile regression are that meteorological ensembles can be used, but they are not needed. A further advantage of the quantile based method is that the uncertainty is explained by a regression like type of model. This gives a possibility for identifying and explaining under which conditions the uncertainty is large or small, respectively.

Combined forecasting

It has previously been shown that the potentials by using more that one provider of the meteorological forecast is very promising. In general an improvement in the interval 5 to 15 % is seen. This improvement is generally obtained by a statistical estimation of the optimal weight which should be put on the individual meteorological forecasts in the combined forecast.

Previously the weights used in the combined forecasts have been fixed over time. In this project the potentials of having a time varying weights or weights which varies as a function of some other forecasts is studied. So far only a rather limited amount of data is considered, and the observed further improvement is only in the order of 1-2 % . However, the methods are very promising, and in cases with more data we might be able to get a further improvement.

The analysis has shown that the weights in the combined forecasts must depend on the air temperature and the turbulent kinetic energy.

1 Introduction

Forecasts of wind power generation are more and more frequently used in various management tasks related to integration of wind generation in power systems. The quality of the forecast is very important, and a reliable estimate of the uncertainty of the forecast is known to be essential. Today the forecasts of wind power generation are provided without a proper consideration to the spatial-temporal dependencies observed in the wind power generation field. The state-of-art prediction systems typically provide forecasts for a single wind farm or a larger region with a number of wind turbines or wind farms.

The purpose of the project was to consider some methods for improving the existing tools for wind power prediction. One of the most important subjects was to consider the spatial-temporal correlation of the prediction errors from the state-of-art prediction system, WPPT (Wind Power Prediction Tool). Another important purpose of the project was to develop methods for calculating reliable estimates of the uncertainty of the wind power production. Finally, methods for improving the forecasts of wind energy by using a combination of several meteorological forecasts is considered. This report summarizes the findings from the various part of the project.

2 Wind Power Prediction Tool

During the entire project the Wind Power Prediction Tools has been used for generating the wind power forecasts. The subject of this section is briefly to describe the main part of this tool. A more complete description can be found in (Madsen et al., 2005) where also some typical configuration examples are shown.

Since version 3 of WPPT the direction dependent power curve modelling of WPPT has been based on conditional parametric models (Cleveland, 1994). In such models the response y_t at time t is modeled using two groups of explanatory variables. One group of variables $\mathbf{x}_t$ enter globally through coefficients depending on the other group of variables $\mathbf{u}_t$, i.e.

$$y_t = \mathbf{x}_t^T \boldsymbol{\theta}(\mathbf{u}_t) + e_t, \tag{1}$$

where $\boldsymbol{\theta}(\cdot)$ is a vector of coefficient-functions to be estimated and e_t is the noise term.

Depending on how the software is set up, the power curve directly models the relation between the meteorological forecast and the power production in a wind farm or in a region. In general the predictions in a region is found by up-scaling the single wind farm predictions, and one of the major purposes of this project is to study the potentials in taking into account the spatial-temporal correlation of the prediction errors.

To account for autocorrelation and diurnal variation WPPT use the output from the conditional parametric model as input to an ARX-model (Madsen, 2007) which adjusts for these effects. These models can be written in the form

$$y_t = \mathbf{x}_t^T \boldsymbol{\theta} + e_t \ , \tag{2}$$

where t is the time index, y_t is the output, $\mathbf{x}_t$ is a vector containing inputs and possibly lagged values of the output, $\boldsymbol{\theta}$ is a vector containing the coefficients of the model, and e_t is the error term.

All models in WPPT are automatically re-calibrated as new information becomes available. For ARX-models adaptive recursive least squares (Madsen, 2007) is used, whereas for conditional parametric models the method described in (Nielsen et al., 2000) is used. Further information about WPPT can be obtained via `www.risoe.dk/zephyr/` or `www.enfor.dk/`.

3 Spatial-temperal corrections

The reason for the spatial-temporal errors is in general due to what is often called phase errors in the meteorological forecasts. In the project a number of different methods for modelling the spatial-temporal errors of WPPT have been considered.

Basically two different methods have been considered. In the first study rigid transformations of the phase errors are considered as a possibility for correcting for the observed errors. In a subsequent study non-linear stochastic models between prediction errors at different geographical locations are considered.

3.1 Rigid transformations of NWPs

Numerical Weather Predictions (NWPs) are used to predict the power production from wind farms. Occasionally changes in wind speed and direction, e.g. arrival of a weather front, occur a few hours before or after the predicted behavior. Assuming that the error is a temporal phase error it can be adjusted for by either a temporary geographic translation or a temporary shift in time. The first approach requires estimation of a 2D translation whereas the latter only requires a 1D estimation.

The aim in this part of the study was to detect and adjust for temporary phase differences between NWPs and the actual weather. The adjustment can either be a spatial or a temporal shift of the NWP.

The methods are described in the research report Christiansen and Nielsen (2006). The conclusions are that only minor improvements (less than 1 pct.) are seen by a rigid transformation of the prediction errors. The concept of shifting NWPs in time can be been used to improve one step predictions. Predictions two steps ahead gave a small improvement and predictions further ahead in time did not improve the predictions. Shifts have been made for the whole Jutland and Funen region and for five groups of wind farms and only the latter resulted in a usable improvement. Still this is too low compared with the autoregresive terms already used in WPPT.

It was the intention that shifts should be used to correct when weather fronts arrive a couple of hours earlier or later than expected but cases where such an error spreads to the whole region was not detected. The reason is most likely that weather changes act as a non-rigid phenomena and therefor simple translations can not be used for correcting for the phase errors.

3.2 Stochastic spatial-temporal model based corrections

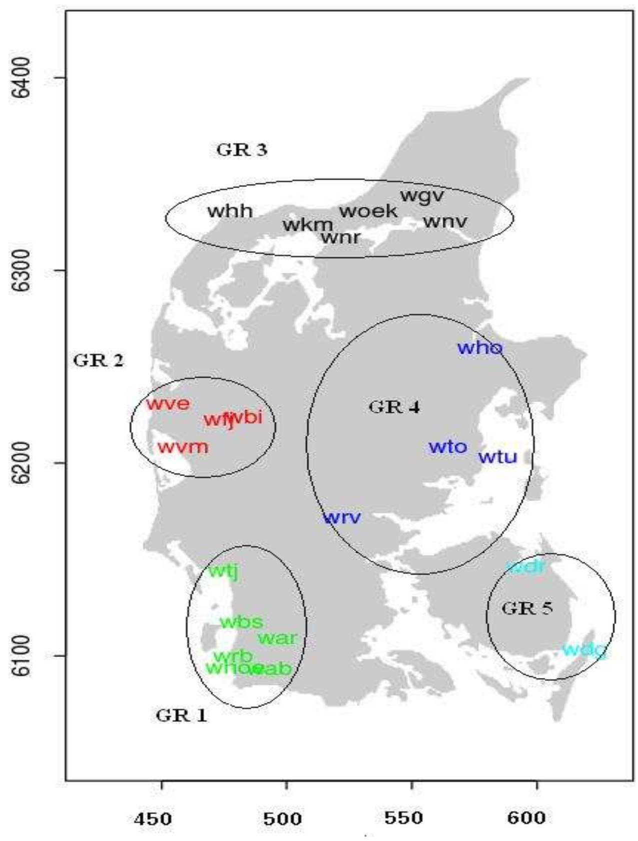

Figure 1: Selected groups of wind farms

Stochastic non-linear corrections offers a non-rigid model between prediction errors at different geographical locations, and, as it will be discussed in the following, this method shows that there is a large potential for improving wind power predictions by considering observed prediction errors at up-stream wind farms or regions. This part of the project is extensively described in the report Kotwa et al. (2007).

In this part of the project new models and methods for improving on-line short-term predictions of wind power were derived and examined. The study was focused on the improvement of the one-hour wind power predictions. The reason is that the data used is from the Jutland and Fuen area, and the typical time-delay in the weather systems from the Western part of Jutland to Fuen is only about 2-3 hours. However, the methodology used in the analysis, could be applied for a longer-term predictions in the similar manner provided that data for a large geographical area is available.

The data selected for this work comes from 24 wind farms owned by Energinet.dk (previously ELSAM) where Wind Power Prediction Tool (WPPT) is used to make forecasts of the power production based on information from the Danish Meteorological Institute (DMI) HIRLAM model.

The location of the considered windfarms and groups is given in the Figure 1. However, only data from 22 farms is used. The remaining 2 farms were excluded at this point of managing the data since a correlation study showed that they can not be reasonably pooled into the same groups together with the other farms.

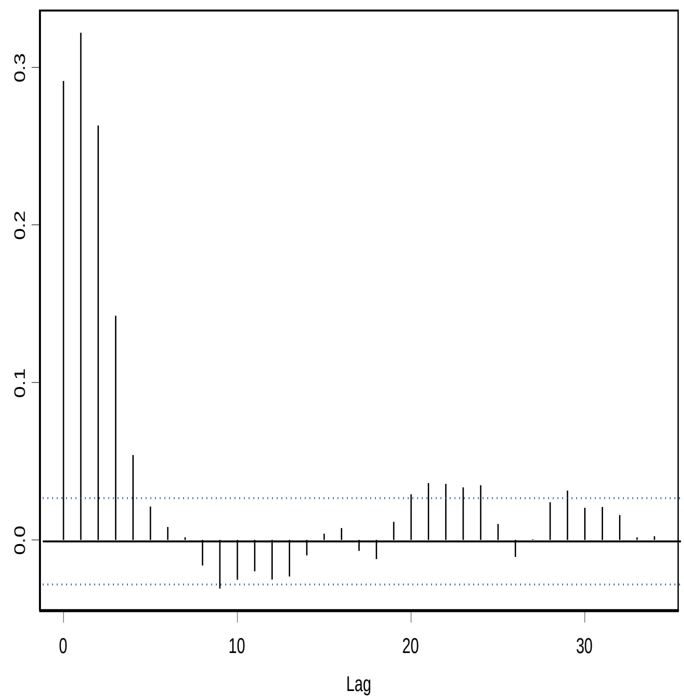

Figure 2: Cross Correlation Function between Group 1 and 5

The errors in groups were calculated as a mean of the errors inside the groups. In the same way the group wind speed was calculated. For the directions in groups the geometrical approach was chosen.

Figure 2 shows the cross correlation function for the mean of errors in Group 1 and Group 5. A rather high cross correlation is seen at lag 1, 2 and 3, the maximum beeing at lag 2. This clearly indicates that if an error is observed in Group 1 at time t then approximately the same error will appear in Group 5 at time $t+2$ on average. Also between Group 4 and 5 a very large cross correlation is seen (not shown, however), but here the maximum cross correlation is observed in lag 1, which seems to be reasonable due to a shorter distance between Group 4 and 5.

One of the most promising models was a model belonging to the class of regime models which enables a possibility of what we call *directional correlation* and *directional regression*. The main purpose is to discover and capture the dependency between wind direction and propagation of prediction errors among the groups. The idea is supported by the intuitive, physical knowledge. We claim that if the wind direction is compatible with the direction of the vector, having its beginning in the group A and ending in B, then the error dependency in those groups should be higher than in case of different directions.

Regime models (or sometimes called threshold models) extend the idea of linear models by letting coefficients vary among regimes. Regimes are defined by the threshold values, which are the upper bounds of the intervals in which the given 'sub-model' is active. The switch of the regimes in threshold models can be governed by previous values of the dependent variable, external signals or unobservable stochastic processes. In this paper we consider the second type of the models.

Define intervals $R_1 \cup ... \cup R_k = \Re$ such that $R_i \cap R_j = \emptyset, i \neq j$. Each interval is given by $R_i = (r_{i-1}, r_i]$. The values $r_0, ..., r_k$ are called thresholds (see e.g Madsen and Holst (2001)). In general the threshold values are to be estimated from the data, but in this report we consider the case when those values are known in advance. The motivation for such an assumption is that we analyze the case when the regimes are governed by the wind directions. Then by applying physical knowledge and intuition it is not difficult to form the regimes. The general form of the models examined further is

$$Y_t = \beta_0^{(J_t)} + \sum_{l \in L_y^{(J_t)}} \beta_l^{(J_t)} Y_{t-l} + \sum_{i \in G^{(J_t)}} \sum_{j \in L_{x_i}^{(J_t)}} \beta_{i,j}^{(J_t)} X_{i,t-j} \qquad (3)$$

where

$$J_t = \begin{cases} 1 & U_t \in R_1 \\ 2 & U_t \in R_2 \\ \vdots & \\ k & U_t \in R_k \end{cases}$$

where U_t is an external signal which determines the regime switch, t is the time index, Y_t is the output variable, X_t are input variables, $\{\epsilon_t\}$ is zero mean white noise, L_y and L_{x_i} are sets of non-negative integers defining the auto-regressive and input lags in the model, G is a set of positive integers defining which input variables to include into the model, J_t indicates the regime and, finally, $\beta_{j,i}$ are coefficients to be estimated.

Since the thresholds are known, the estimation problem is solved by fitting different linear models to the data in each of the regimes. The technique used for estimation is again Least Squares, as described in the previous section.

Let us assign the sequence of the forecasted wind direction at the Group 5 as U_t. The time series Y_t is divided according to U_t values into 4 groups. The structure of the regimes is shown below:

$$J_t = \begin{cases} 1 & for \quad U_t \in (0, 90] \text{ (North-East wind)} \\ 2 & for \quad U_t \in (90, 180] \text{ (South-East wind)} \\ 3 & for \quad U_t \in (180, 270] \text{ (South-West wind)} \\ 4 & for \quad U_t \in (270, 360] \text{ (North-West wind)} \end{cases}$$

The decision of fixing the threshold values was dictated by easiness in interpreting the influence of wind direction which in this case is compatible with the geographical cardinal directions. Furthermore, other divisions were checked and the improvement in model fit was considered insignificant or none.

As a result we obtain a four-regime model determined by the external signal (forecasted wind direction for time t). Linear models are fitted for each of the regimes. The optimal number of lags is decided separately for each regime as it was done in the previous section. Table 3.2 shows the structure of the final threshold model.

J_t	Lags in Group 1	Group 2	Group 3	Group 4	AR
1	-	-	4th	1st	10
2	1st	-	-	1st	5
3	1st, 2nd, 4th	-	-	1st	6
4	1st and 3rd	-	-	1st	6

Table 1: Threshold model structure

The choice of the variables seems to be reasonable if the position of the farm groups is taken into account (see Figure 1), e.g. for the direction (270,360] which corresponds to the northeast wind, influence of Groups 1 and 4 is seen to be most significant. Maximum lags taken for Groups 1 and 4 conform with the *directional distance* from Group 5 in this regime.

The performance of the Threshold model is shown in Table 2.

The overall R^2 of the Threshold model is 0.4991

In general the results of the modelling work showed a large potential in improving the

J_t	R^2	RMSE
1	0.3840	0.05763
2	0.4614	0.04352
3	0.4932	0.06832
4	**0.5490**	0.05579
overall model	**0.4991**	0.05742

Table 2: Threshold model results

WPPT by modelling the spatial-temporal correlation of the errors between the groups. The captured error propagation appears to be dependent on the forecasted weather situation (mainly wind direction) and geographical position of the wind farms. Methods applied in the project captured a non-linear behavior of error dependency.

The regime model described above is further improved by considering also the forecasted wind speed. Hence, during the study more complex models were fitted to data. Varying coefficient models were fitted in order to capture the dependency on the forecasted wind speed level and the time since the last weather forecast was obtained. However, the results indicated that the improvement by also including the wind speed in a non-linear model are minor.

On average the model adequately explains more than 49% of the error variation for one-hour predictions, and if only directions where upstream data is available is considered, the model explains more than 54% of the error variation.

4 Adaptive quantile regression

Today a reliable knowledge about the uncertainty of the wind power prediction is essential. It has been shown in Pinson et al. (2007) that the value of using an advanced prediction tool for wind power prediction for the Dutch marked leads to 38% savings on trading cost, but if also a reliable estimate of the uncertainty of the future wind power is available a further saving of 39% of the trading cost is obtained. Hence, the value of knowing the uncertainty might be even higher than the value of having an advanced prediction tool.

For wind power it must be realized that the uncertainty or the random behavior of the future wind power production behaves in a rather complicated way on the e.g. the expected wind power prediction.

One way of obtaining information on the random behavior of the prediction errors is to estimate higher order moments of the data. If some assumptions are made on the distribution of errors, then a finite number of moments will characterize the distribution. For example if the error is assumed to be Gaussian then the two first moments characterize the distribution completely. This method has been used some years ago in previous implementations of WPPT.

Another possibility of obtaining information about the uncertainty is to used statistically calibrated ensemble based estimates of the quantiles of the wind power production for the next hours. This procedure has been developed a few years ago and is described in a number of papers and reports – see e.g. Nielsen et al. (2007a) and Nielsen et al. (2004). This method is successfully used in WPPT-E.

As indicated it is not reasonable to assume a specific known distribution of the wind power data data. For this reason quantile regression is developed in this project as a new way of estimating quantiles directly using regression based techniques. In addition to the distribution of data being unknown, it is often reasonable to assume that the distribution of data vary over time. Hence the most well suited method calls for an adaptive estimation of the quantile regression parameters.

The loss function for quantile regression is based on a weighting of the absolute values of the residuals. Such problems can be solved using linear optimization techniques.

The existing methods for finding solving the optimization problem related to quantile regression do not offer the possibility to use knowledge about the solution of the system. In this project methods or algorithms where knowledge about the solution at time t are used to solve the system at time $t+1$, and thereby reducing the computational effort and increase computation speed.

A new method is suggested and described in the article Møller et al. (2008). This method is found to be well suited for solving the time varying problem related to estimating the uncertainty of future wind power predictions. In this article we present the linear optimization formulations of the quantile regression problem and proposes a solution using the simplex method directly, which enables a formulation that is suited for a computer implementation. The implementation of the simplex method is then combined with an updating procedure for updating information related to the actual estimates.

The effect of the explanatory variables is described by by using splines basis functions. In online settings for wind power prediction data are often arrive every 15th minute making the possibility to update the solutions at each of these time points. The strength of the methods are illustrated in the article, where the time adaptive method prove to be superior to corresponding static models.

4.1 Quantile regression

Let us briefly describe the method of quantile regression.

Quantile regression as presented by Møller et al. (2008), is based on a linear model

$$y = \boldsymbol{x}^T \hat{\boldsymbol{\beta}} + r = \hat{Q}(\tau; \boldsymbol{x}) + r \tag{4}$$

which is fitted using an asymmetrical and piecewise linear loss function

$$\rho_\tau(r) = \begin{cases} \tau r & , \quad r \geq 0 \\ (\tau - 1) r & , \quad r < 0 \end{cases} \tag{5}$$

The observation equations are written as

$$\boldsymbol{y} = X \hat{\boldsymbol{\beta}} + \boldsymbol{r} \tag{6}$$

X is called the design matrix, $\hat{\boldsymbol{\beta}}$ is the coefficients to be estimated and $\boldsymbol{y}$ is the response from the system. The estimate of $\boldsymbol{\beta} \in \mathbb{R}^K$ given N observations is

$$\hat{\boldsymbol{\beta}}(\tau) = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \rho_\tau(r_i) = \arg\min_{\boldsymbol{\beta}} S_\tau(\boldsymbol{\beta}; \mathbf{r}) \tag{7}$$

This is a linear programming problem. A solution to Eq. (7) is

$$\hat{\boldsymbol{\beta}}(\tau) = X(h)^{-1} \boldsymbol{y}(h) \tag{8}$$

where h is a K element index set from the set $\Omega = \{1, 2, ..., N\}$. $X(h)$ refer to rows in X. Solutions like this is obtained using linear programming techniques. In general interior point methods are considered superior to the simplex algorithm for large problems.

The proposed algorithm is based on the simplex method. We use the simplex algorithm directly to update the solution as new observations become available. The structure of the quantile regression problem is exploited to make an efficient implementation of the simplex algorithm.

4.2 An example

Figure 3 shows a 7 days ahead forecast of the wind power production with reliable quantiles. It is clearly that the uncertainty varies in time in a rather complicated manner. In the figure only the 25% and the 75% quantile are shown.

The difference between the called the Inter Quantile Range (IQR), and an evaluation of the developed method for quantile regression based estimation of the uncertainty shows that the most important variable for predicting the uncertainty is the predicted wind power. Some attempts have been taken to use several meteorological forecasts also in improving the uncertainty of the wind power prediction. The results have shown so far that a difference between two independent providers of the meteorological forecasts contains additional valuable information about the uncertainty (see Møller et al. (2008)).

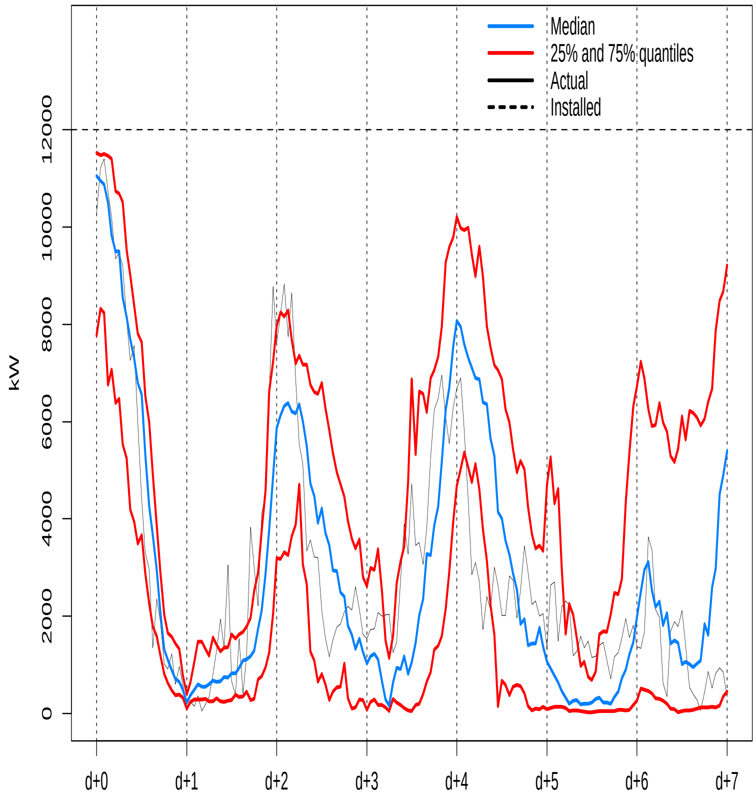

Figure 3: An example of a 7 days ahead prediction with reliable quantiles

5 Combining meteorological forecasts

It has been shown previously that the use of use of multiple meteorological forecasts will, in particular if the forecasts come from different suppliers, result in operational robustness and a rather large improvement of the precision of the predictions. The result is obtained by running multiple instances of the forecasting software, e.g. WPPT, and thereby producing multiple wind power forecasts.

It has been observed that the by combining meteorological forecasts from two to tree independent provides leads to an improvement of 5 to 15% compared to the best individual forecast.

In fact this is the range of improvements which is obtained. For Klim an overall level (across horizons) of 9% is obtained. These results are obtained when combining three individual forecasts based on meteorological models nested in different global models.

Based on the investigations in (Nielsen et al., 2007b) it is recommended that two or three good meteorological forecasts are used and the forecast errors of these should have low correlation (less that approximately 0.8). This seems to be the case for meteorological forecasts originating from different global models.

In this project it is studied how the weight in the combined forecasts varies as a function of time and as a function of other explanatory variables. In general it is shown that the optimal weights should be selected as a function of the meteorological forecasts in order to obtain the optimal time varying weights. The results are described in detail in the project report Thordarson et al. (2007). In particular, it is concluded that the optimal weights depends on the air density and the turbulent kinetic energy.

The data used in the analysis was twofold, data set including three wind power predictions from WPPT and data set of MET forecasts which were used to estimate weights in a combined forecast. Both data sets contained forecasts at time point 00Z, midnight, for the next 24 hours ahead. There were 7272 data points in the data sets which span the period from February 2nd, 2003 to December 2nd, 2003. The data set with the power forecasts consisted of measurements of power production from Klim wind farm and the predicted power from WPPT, based on three different weather forecasting systems, i.e. DWD, DMI-HIRLAM and MM5. The second data set included of MET forecasts from DMI-HIRLAM, consisting of, e.g., wind speed, wind direction, air density, turbulent kinetic energy and radiation. Many of the variables were further divided with respect to atmospheric pressure levels.

Since the problem of combining was first presented many methods have been developed. However, the adoption of the linear model has been the most favorable when combining. Applied to wind power prediction the variable of interest, the predicted variable, is the actual wind power production and at time t it is denoted as y_t. Let $\widehat{y}_{i,t}$ be the i-th individual forecast at time t, the prediction error between the production and the i-th competing prediction is $e_{i,t} = y_t - \widehat{y}_{i,t}$ where $i = 1, \ldots, K$. A linear combination of the forecasts is formulated as

$$y_t = \widehat{y}_{c,t} + e_{c,t} = w_{0,t} + \sum_{i=1}^{K} w_{i,t}\widehat{y}_{i,t} + e_{c,t}. \tag{9}$$

where $w_{i,t}$ is the weight given to forecast i at time t. The term $w_{0,t}$ represents the constant in the linear model but inclusion of intercept is assumed to reflect the bias of the individual forecasts. The prediction error for the combination is then given by $e_{c,t} = y_t - \widehat{y}_{c,t}$. Further, the weights are restricted to sum to one, i.e. $\sum_{i=1}^{K} w_{i,t} = 1$.

The methods applied were the recursive least squares method (along with static least squares method) and the adaptation of minimum variance-covariance. The performance of the simple average method was also applied for comparison. Both constant and restriction in regression method was used. Counting for the restriction is equal to use the optimal procedure in combining but including the intercept takes out the bias of the individual forecasts. The relation between the Recursive Least Square (RLS) and the Minimum Variance (MV) methods was analysed, and showed simply that the intercept in the model can not be neglected.

The RLS method gave a significantly better performance compared to the MV method. Further, the processes through all 24 prediction horizons were parallel with RLS closer to the measurements. The analysis showed also that the simple average method was the worst performing method for combination. The performance in the study is measured by insample Root Mean Square Error (RMSE) and coefficient of determination (R^2).

The objective in the study was to extend the combination model in (9) such as the weights were now functions of some of the MET forecasts. Inspecting the MET variables by using coplots (the dependent variable analysed as a function of two explanatory variables) the air density and turbulent kinetic energy at pressure level 31 were extracted, as the weights showed some trend as a function of these two MET forecasts. The performance for the conditional parametric model was compared with the LS and RLS method, which showed to be significantly outperforming the MV method.

Compared to the LS method the methods are performing quite alike for the first 12 prediction horizons except DWD/MM5. That specific forecasts is very different than the competing performance for the first 8 horizons, but thereof it has lower RMSE. For larger horizons forecasts using MET variables are outperforming the offline model, estimated over the entire data set. **On average the improvement was 1%, but considering not the shorter prediction horizons this improvement is closer to 2%**.

With MET based forecasts outperforming the offline performance for large horizons it is interesting to compare it with the RLS performance. This comparison reveals that over the intermediate prediction horizons the MET dependent forecasts are very close to the RLS performance. For small horizons all the combined forecasts from RLS are significantly better. The difference is at its most for the shorter horizons but then decreases until the intermediate horizons. For the larger horizons both DWD/HIRLAM and DWD/MM5 forecasts are performing similarly, but the difference between the MET dependent HIRLAM/MM5 forecast and corresponding RLS forecast increases. The performance of the conditional parametric model, where the weight were only a function of the turbulent kinetic energy, appeared to be approaching the offline performance but with a slight improvement for the larger prediction horizons.

As described in the report (Thordarson et al. (2007)) there are few things which can be further elevated. The behavior of the combined forecasts where MET variables are used to generate weights is better than forecasts with parameters estimated for the whole data set. By estimating the weights adaptively extensive improvement is expected. Combining more than two forecasts by using MET variables is also something that can be considered. In comparison of the wind power forecasts it is concluded that combining all three wind power forecasts give the most accurate combination. The third forecast is not deemed in the analysis of the MET data but is an interesting topic for further analysis to improve combined forecasts using MET variables. The MET forecasts used in the study are only the ones from *DMI-HIRLAM*. Three power forecast models are used in the analysis which all use different MET data as input. Applying more MET forecasts to the analysis gives the researcher a lot more information to work with since the inputs for the weight estimation are the MET forecasts.

6 Conclusion

This PSO project is designed as a pilot study for investigating potential areas where an improved wind power prediction can be obtained. Furthermore, a new methods for estimating the uncertainty of the wind power prediction is presented.

Considering the improvements of wind power predictions it is that the very large improvements are seen for stochastic non-linear models for the spatial-temperal correlation in the farm or group specific prediction errors. In general about 50 % of the one-hour prediction error can be explained by the suggested non-linear model. The most promising model is a non-linear regime models, where the regime is determined by the predicted wind direction.

It is also demonstrated that a rigid transformation of the prediction errors in order to account for phase errors of the meteorological prediction only leads to a minor improvement of the wind power predictions.

Some resent results have shown that the value of having a reliable estimates of the uncertainty is of the same order as the value of have a high quality (state-of-art) prediction system like WPPT.

In this project a new and very promising method for estimating the uncertainty for wind power predictions is developed. The methods is called **Adaptive Quantile Regression**, and it is seen that this approach can be liked to the WPPT forecasts rather easily, and hereby a reliable estimated of the uncertainty of the wind power prediction will be obtained.

It has previously been shown that the potentials by using more that one provider of the meteorological forecast is very promising. In general an improvement of in the interval 5 to 15 % is seen. This improvement is generally obtained by a statistical estimation of the optimal weight on the individual meteorological forecasts. In this project the variation in time of the weights is studied, and the observed further improvement is only in the order of 1-2 % . However, the methods are promising, and in cases with more data we might be able to get a further improvement. The analysis has shown that the weights in the combined forecasts must depend on the air temperature and the turbulent kinetic energy.

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References

Christiansen L. and Nielsen T. Shifting nwps in time to improve short time predictions. Pso-fu5766, IMM/DTU, 2006.

Cleveland W. *Multivariate Analysis and Its Applications*, chapter Coplots, Nonparametric Regression, and conditionally Parametric Fits, pp. 21–36. Institute of Mathematical Statistics, Hayward, 1994.

Kotwa E., Nielsen H., Madsen H., and Vlasova J. Spatio-temporal modelling of short-term wind power prediction errors. PSO-FU5766 18, IMM/DTU, 2007.

Madsen H. *Time Series Analysis*. Chapman and Hall, 2007.

Madsen H. and Holst J. *Non-linear and non-stationary time series analysis*. DTU and University of Lund, 2001.

Madsen H., Nielsen H.A., and Nielsen T.S. A tool for predicting the wind power production of off-shore wind plants. In *Proceedings of the Copenhagen Offshore Wind Conference & Exhibition*. Danish Wind Industry Association, Copenhagen, 2005. `http://www.windpower.org/en/core.htm`.

Møller J., Aa. Nielsen H., and Madsen H. Algorithms for adaptive quantile regression - and a matlab implementation. Pso-fu5766, IMM/DTU, 2007a.

Møller J., Nielsen H., and Madsen H. Combined forecast and quantile regression. Pso fu-5766, IMM/DTU, Lyngby0, 2007b.

Møller J., Nielsen H., and Madsen H. Time-adaptive quantile regression. *Computational Statistics and Data Analysis*, **52**:1292–1303, 2008.

Nielsen H., Madsen H., Nielsen T., Badger J., Giebel G., Landberg L., Sattler K., and Feddersen H. Wind power ensemble forecasting. In *Proceedings of the 2004 Global Wind Power Conference and Exhibition*. Chicago, 2004.

Nielsen H., Nielsen T., Madsen H., Pindado M., and Marti I. Optimal combination of wind power forecasts. *Wind Energy*, **10**:471–482, 2007a.

Nielsen H.A., Nielsen T.S., Joensen A.K., Madsen H., and Holst J. Tracking time-varying coefficient-functions. *Int. J. of Adaptive Control and Signal Processing*, **14**(8):813–828, 2000.

Nielsen H.A., Nielsen T.S., Madsen H., San Isidro M.J., and Marti I. Optimal combination of wind power forecasts. *Wind Energy*, 2007b. Accepted, `http://www.imm.dtu.dk/~han/pub/FU4101_WEpaperCFCaccepted.pdf`.

Pinson P., Chevallier C., and Kariniotakis. Trading wind generation with short-term probabilistic forecasts of wind power. *IEEE Transaction on Power Systems*, **22**:1148–1156, 2007.

Thordarson F.O., Madsen H., and Aa. Nielsen H. Optimal combined wind power forecasts using exogeneous variabls. PSO-FU5766 17, IMM/DTU, 2007.